AMATH 569 Final Examination Solution

June 7, 2021. 2:30-4:30 pm, Seattle time. Must be uploaded to Canvas by 4:35pm. Open book and notes. Work on your own. No online searches. No computer apps such as Mathematica, Wolffram Alpha or Maple. 100 points total.

**1.**  (a) **(10 points)** Solve the one-dimension wave equation using any method you prefer:

PDE: 

BC: 

IC: 

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Use D’Alembert solutions:



The first initial condition implies that  The second initial condition implies that  So



(b) **(10 points)** Relate the above problem to the Green’s function defined by:

PDE: 

BC: 

IC: 

And write down the solution for *G*. You can make use of the solution in (a) if you wish.

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For  the PDE is the same as in (a).

For 

These are the same as the initial conditions in (a) except with 



(c) **(10 points)** Use the result in (b) to solve:

PDE: 

BC: 

IC: 

where the forcing term *Q* is given by: 

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**2. Short answers:**

**(a) (10 points)** If *f(x)* is integrable, i.e.  is finite, for what values of is its Fourier transform  defined? Write down the inverse Fourier transform.

Real values of 



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**(b) (10 points)** If *f(x)* is not integrable, with  for what values of *s* is its Laplace transform  defined? Assume  Write down the inverse Laplace transform.

Real part of s>b



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**(c)** **(10 points)** In trying to solve the well-posed problem at large but finite *t:*

PDE: 

BC: 

IC: 

Assume that the forcing is persistent so that for large *t* one can ignore the initial conditions and we assume the solution to be of the form: 

Does there exist a solution satisfying the problem that results:



 ?

*k* is real*.*

If your answer is yes, then obtain that solution*.* If your answer is no, explain why not.

No, there is no solution that satisfies the boundary condition . The solution behaves like 

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**(d) (10 points)** Add a small damping to the problem in (c). Consider the PDE



subject to the boundary condition:



Assume:



Does the solution exist for ? If yes, obtain that solution. If no, explain why not.

Solved in Notes.

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**3. (30 points)** Solve the following initial-value problem for 



Implicit solution is fine. Do shocks form? If so, when?

You do not have to solve for the solution past the shock formation time.



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